MATH 8: HANDOUT 3
COMBINATORICS REVIEW PART 1

Your solutions should include explanations allowing me to see how you arrived at the answer.

Multiplication Principle

Let us suppose that a task comprises of multiple random experiments that can be completed

- n_1 outcomes of a random experiment E_1
- n_2 outcomes of a random experiment $E_2 \cdots$
- n_k outcomes of a random experiment E_k

then if each experiment E_i can be completed in n_i different ways and the E_{i+1} can be completed in n_{i+1} different ways after the previous experiments are completed, then the total number of ways of completing the composite experiment is $n_1 \cdot n_2 \cdot \cdots \cdot n_k$.

COMBINATORICS CLASSWORK

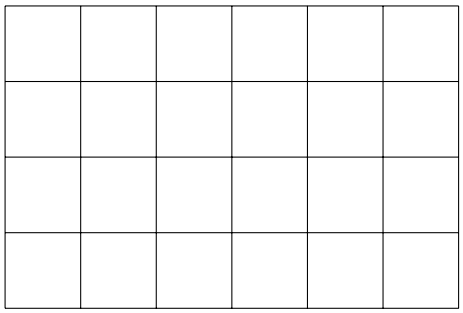
1. In how many ways can 20 people fill four distinct executive positions, President, Vice President, Treasurer and Secretary?
2. A club consisting of 20 people need to choose one President and three Council Members. In how many ways can they do this?
3. How many words one can get by permuting letters of the word "simple"? of the word "DANDANA"?
4. How many possible license plates could be stamped if each license plate were required to have exactly 3 letters and 4 numbers?
5. Now, how many possible license plates could be stamped if each license plate were required to have 3 unique letters and 4 unique numbers?
6. How many subsets are possible out of a set of 4 elements? What about 20 elements?
7. In a meeting of 4 people, every one of them shakes hands once with every other. How many handshakes was it altogether? What about a meeting with n=100 people?
8. There is a round table seating 8. How many ways there are for 8 people to choose their seats at the table? What if we do not distinguish between two seatings which only differ by rotating the table?

HOMEWORK

Combinatorics:

1. There are four sheep in a pen, each with a distinct color of wool. I want to make a sweater from their wool (I will ask the sheep nicely before shearing them).
   (a) If I want a sweater with two colors of wool, how many possible pairs of colors are there for me to select from?
   (b) If I want a sweater primarily of one color with a trim made of a second color, how many possible ways are there for me to pick these two colors?
2. 12 sentient frogs wish to select 1 leader and then a board of administration. The board is to be comprised of 3 frogs, and the leader may not be on the board. How many possible ways are there to fill the positions? (Assume all the frogs are distinct, with distinct personalities).
3. How many ways are there to select two black cards and three red cards from a (standard) deck of cards? (In a standard deck there are 52 cards, with exactly half red and the other half black).
4. How many two-digit numbers are there where the first digit is strictly larger than the second digit? Examples: 42, 61, and 10 are such integers, but 18, 44, and 56 are not. Hint: Casework.

5. 15 students come to a classroom with 25 seats. How many ways are there of seating these students?
6. 15 students play musical chairs with 10 chairs (some of them cannot be sited). How many ways are there of seating these students during the first round of the game?
7. How many words one can get by permuting letters of the word "tiger"? of the word "rabbit"? of the word "common"? of the word "Mississippi"?
8. There are 4 different beads available for a necklace. In how many ways can a designer create a necklace? What about 10 distinct beads?
9. A ship's captain sends signals by arranging 3 blue and 5 red flags and 2 white horizontal flags on a vertical pole. How many different signals could the ship's captain possibly send?
10. How many paths are there from start to end on a 6×4 grid as shown in the picture? The path should always be going to the right or up, never to the left or down.



Continued Review of Algebra.

1. Let $x + y = 17$ and $xy = 52$
   (a) Calculate $(x + y)^2$.
   (b) Calculate $\frac{1}{x} + \frac{1}{y}$.
   (c) Calculate $(x + y)^3$.
2. Let x_1 and x_2 be the solutions of $x^2 - 8x - 33 = 0$. Without solving the equation find the value of the expression below(Hint: write the expression in terms of $x_1 + x_2$ and x_1x_2)
$$x_1(1 + x_1) + x_2(1 + x_2).$$
3. Solve the following inequality. Write your answer as a set of possible values for x.
$$\frac{(x - 3)(2x + 1)}{(x + 17)} \geq 0$$
4. Solve the equation:
$$|2x - 5| < 10$$
5. Let a, b, c be distinct positive integers. Is it possible that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$? How about $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$?
6. Let n be a positive integer greater than 10. Is it possible for n to have more than $\frac{n}{2}$ factors?
7. Determine the distance between the two intersection points of the graphs of $y = x^2$ and $y = x + 2$.
8. Write the square root of 32 in simplified form as $a\sqrt{b}$ for positive integers a and b (*simplified* means a should be as large as possible).