

MATH 8: HANDOUT 10
LOGIC 4: LOGIC PROOFS CONTINUED

WARM-UP

Find the logical conclusion of the statements below:

People allergic to bananas are also allergic to oranges. Who is allergic to apples is not allergic to oranges.

HOMEWORK

1. Use truth tables to check if:
 - (a) the following statements are equivalent: "If you do not clean up your room, you will be punished" and "Clean up your room, or you will be punished".
 - (b) the following statements are equivalent: "If a student does not take the final exam, that student gets an F", "If the student does not get an F, that student must have taken the final exam"
 - (c) $A \implies B$ is equivalent to $(\neg A) \vee B$
 - (d) $A \implies B$ is equivalent to $(\neg B) \implies (\neg A)$ (the last statement is called a *contrapositive*).
 - (e) $(\neg(A \implies B)) \iff (A \wedge (\neg B))$ (i.e. the *contradiction* of $A \implies B$).
2. Now let us neatly apply what we learned about the "if-then", i.e. $A \implies B$ statements for simple proofs:
 - (a) Try to prove using the contrapositive that "if n is an integer and $5n + 2$ is even, then n is even"
 - (b) Try to prove using the contradiction that "if n is an integer and $5n + 2$ is even, then n is even"
3. Let A and B be the propositions "The election is decided" and respectively "The votes have been counted". Write each of the following logical propositions as an English sentence.
 - (a) $\neg A$
 - (b) $(\neg A) \wedge B$
 - (c) $(\neg B) \implies (\neg A)$
 - (d) $(\neg B) \vee ((\neg A) \wedge B)$
4. For each of the following statements, write an English statement which describes the negation of that statement in the most direct way.
 - (a) This week it will snow on Wednesday.
 - (b) This week it will snow on Saturday and Wednesday.
 - (c) If it snows on Saturday this week then it will not snow on Wednesday.
5. For each of the following Lewis Carroll puzzles: identify the propositions, express the statements as "if-then", and afterwards state the logical conclusion from the conditionals.
 - (a) All unripe fruit are unwholesome. All these apples are wholesome. No fruit, grown in the shade, is ripe.
 - (b) Colored flowers are always scented. I dislike flowers that are not grown in the open air. No flowers grown in the open air are colorless.
 - (c) None of the unnoticed things, met with at sea, are mermaids. Things entered in the log, as met with at sea, are sure to be worth remembering. I have never met with anything worth remembering, when on a voyage. Things met with at sea, that are noticed, are sure to be recorded in the log.