MATH 8: MATH BATTLE DEC 18, 2022

1. Prove that every perfect square p^2 (with p a natural number) has an odd number of positive factors and that every nonsquare n has an even number of positive factors.

2. How many 7-digit numbers have no two adjacent digits equal?

3. Four points are chosen at random on a grid of 5 by 5 dots. In how many ways can we choose four points such that they are the vertices of a rectangle whose sides are parallel to the sides of the grid?

4. For what real values of x is $(1 - |x|)(1 + x)$ positive ?

5. A rubber ball is dropped from a 100 ft tall building. Each time it bounces, it rises to three-quarters its previous height. So, after its first bounce it rises to 75 ft, and after its second bounce it rises to 3/4 of 75 ft, and so on. What is the total distance the ball travels?

6. What is the greatest odd integer that is a factor of 5! ? What about 7! ?

7. A dart is thrown at the square target shown. Assuming the dart hits the target at a random location, what is the probability that it will be in the shaded region? Express your answer as a common fraction.