

MATH 8
HANDOUT 2: PASCAL'S TRIANGLE

PASCAL TRIANGLE

				1								
				1	1							
				1	2	1						
				1	3	3	1					
				1	4	6	4	1				
				1	5	10	10	5	1			
				1	6	15	20	15	6	1		
				1	7	21	35	35	21	7	1	
				1	8	28	56	70	56	28	8	1

Every entry in this triangle is obtained as the sum of two entries above it. The k -th entry in n -th line is denoted by $\binom{n}{k}$, or by ${}_n C_k$. Note that both n and k are counted from 0, not from 1: for example, $\binom{2}{1} = 2$. Thus, these numbers are defined by these rules:

$$(1) \quad \begin{aligned} {}_n C_0 &= {}_n C_n = 1 \\ {}_n C_k &= {}_{n-1} C_{k-1} + {}_{n-1} C_k \quad \text{for } 1 \leq k \leq n-1 \end{aligned}$$

And these rules are sufficient to define the entire triangle.

These numbers appear in many problems:

- ${}_n C_k$ = The number of paths on the chessboard going k units up and $n - k$ to the right
- = The number of words that can be written using k zeros and $n - k$ ones
- = **The number of ways to choose k items out of n (order doesn't matter)**

PROBLEMS

In this homework assignment (and in all other assignments in this class), many problems are non-trivial and require some thought. Try to start early. You are not expected to be able to solve all of the problems, so do not be discouraged if you can't solve some of them. The solutions are to be written on separate sheets of paper (as neatly as possible), with your name at the top, and submitted by the next class. Please make sure that you write not just the answer but also the solution, i.e. your reasoning showing how you arrived to this answer. Ideally, your solution should be such that someone who doesn't know how to solve this problem can read it and follow your arguments.

It is enough if you can write the answers in terms of factorials and binomial coefficients — it is not necessary to actually compute them: the answer like $13!$ or ${}_{10} C_5$ is good enough.

1. If we want to choose a president, vice-president, and two assistants from a 15-member club, in how many ways can we do it?
2. 5 kids come to a store to choose Halloween costumes. The store sells 25 different costumes. Assuming the store has enough stock for the kids to choose the same costume if they want, in how many ways can the kids choose the costumes? What if they want to choose so that all costumes are different?
3. Suppose I flip a coin three times, and I record its result each time (for example, the coin may land heads then tails then heads, which I will write as HTH , where order matters). I will refer to this three letter combination as the 'final result' - for example, HHH is the only final result that has no tails.
 - (a) How many final results are there with exactly one tail?
 - (b) How many final results are there with exactly two tails?

