April 23, 2023 Math 9

Complex numbers.

Test.

1. Write and prove the de Moivre's formula for a complex number, $z = x + iy = r(\cos\varphi + i\sin\varphi)$,

$$z^n = \left(r(\cos\varphi + i\sin\varphi)\right)^n = \cdots ?$$

2. Write the expression for the n-th root , w, of a complex number, $z = x + iy = r(\cos\varphi + i\sin\varphi)$,

$$w = \sqrt[n]{z} = \cdots ?$$

How many such complex roots are there?

3. Find and draw on a complex plane all roots of the equation,

$$z^5 = 1$$

4. Find all complex numbers z such that:
   a. $z^2 = -i$
   b. $z^4 = 16 - 8i\sqrt{3}$
   c. $z^5 = 1 + i$
5. For the complex number, $z = x + iy = r(\cos\varphi + i\sin\varphi)$, write the Euler's formula for,

$$w = e^z = \cdots ?$$

6. Find all roots of the polynomial $1 + z + z^2 + z^3 + \cdots + z^{n-1}$
7. Without doing the long division, show that $1 + z + z^2 + \cdots + z^{99}$ is divisible by $1 + z + z^2 + \cdots + z^{49}$.
8. Solve the following equation,

$$2\cos^2 \pi x - 3\sin \pi x + 1 = 0$$