Homework for May 14, 2023.

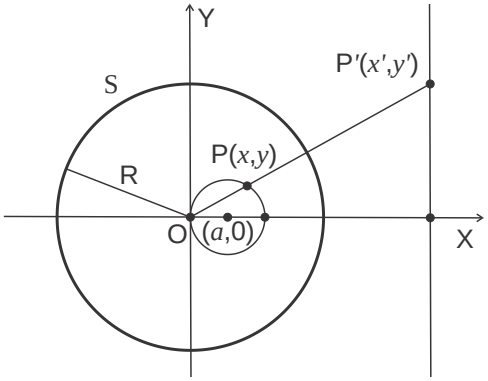
Geometry.

Review the classwork handout on inversion. Solve the unsolved problems from the previous homework. Solve the exercises and the following problems.

Problems.

1. Given circle C and its image C' of find the inversion circle, S, which transforms one into another. Consider three cases:
   a. circles C and C' are crossing, i.e. have two common points
   b. circles C and C' are touching, i.e. have one common point
   c. circles C and C' have no common points
2. Find the distance between two parallel straight lines that are images of the two circles with the radii r_1 and r_2, which are tangent at the center O of the inversion circle S with radius R.
3. Express the similarity coefficient between circle L and its image L' through radius of the inversion circle R and length of the tangent, $|OT|$. What happens if $|OT| = R$?
4. Consider inversion with respect to circle S centered at the origin, (0,0). Image of point $P(x, y)$ is point $P'(x', y')$. Prove that the transformation of coordinates is (see figure),

$$x' = x\frac{R^2}{x^2 + y^2}$$
$$y' = y\frac{R^2}{x^2 + y^2}$$



5. What is the image of the line $y = ax + b$?
6. Show that in the case $a \neq r$ there exist x_0, y_0, r_0, such that the image of circle $(x - a)^2 + y^2 = r^2$ is circle $(x' - x_0)^2 + (y' - y_0)^2 = r_0^2$.