TEMPERATURE

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Theory recap

Introduction. Last time we discussed the relation between work and energy. That concluded our discussion of mechanical energy. We learned that mechanical energy includes kinetic energy and potential energy. We learned that mechanical energy of an object could be changed if some force performs work on it.

Mechanical energy is not the only form of energy. For example, if some objects slides on a surface with friction, kinetic energy of this object decreases. No other object gains speed or moves up, so mechanical energy is certainly decreasing. But energy is never lost completely, it is just transferred to other forms - like in this case to thermal energy. Today we start discussing thermal energy and thermal phenomena. We begin this discussion with a very familiar but also a very interesting concept - temperature.

What is temperature. Every one of us can tell hot from cold by how it feels like, but what does it actually mean that something is hot or cold? Where does this notion originate physically? The fact that temperature can be increased by friction tells us that it has something to do with energy, but energy of what?

You are already familiar with the fact that all objects around us are built out of very small constituents: atoms and molecules. These tiny bits of matter are constantly moving at very high speeds, even if the object as a whole is motionless. They collide with each other and change the direction of their motion very often, so the object as a whole stays at rest. We will discuss this a little further when we talk about phases of matter in several weeks.

But since atoms and molecules constantly move, they have kinetic energy, right? Kinetic energy is present for any object with mass and speed, no matter how tiny it is. Atoms and molecules have mass, even though it is small, and speed, which is in fact not so small, as we are going to see. This kind of kinetic energy that is associated with internal motion of constituents of an object and not with the motion of an object as a whole is called **internal energy**.

Temperature is the measure of internal energy! Basically, the higher temperature is, the faster atoms and molecules move inside an object (so their kinetic energies are higher and so internal energy is bigger). The discovery that temperature that everyone feels is due to constant motion of tiny particles, which comprise everything around us and are invisible to the naked eye, is actually one of the biggest achievements of physics.

Temperature scales. In order to define a scale we need to set up two reference points and assign two different temperatures to them. A convenient choice is to use freezing point of fresh water as one reference point and boiling (at normal atmospheric pressure) point of water as the other reference point. These reference points are particularly easy to reproduce (because almost everywhere and at any time one has access to fresh water) which is important for a widespread use.

For example, in Fahrenheit scale freezing point of fresh water corresponds to 32° F and boiling point corresponds to 212° F. 0° F corresponds to freezing temperature of a particular solution of water and salt. 100° F is slightly above the normal temperature of a human body.

Another scale which is common in countries with metric units is Celsius scale. In Celsius scale freezing point of water is assigned 0° C and boiling point of water - 100° C so these water reference points are really built in it.

If one wants to go between these scales, a conversion formula exists. If t_F is temperature in Fahrenheit and t_C is the corresponding temperature in Celsius, they are related as follows:

$$t_F = 32 + \frac{9}{5} t_C$$

Let us check that reference points are obtained correctly: for 0° C (freezing point of water) this formula gives $32 + 0 = 32°$ F which is correct. For 100° C (boiling point of water) this formula gives $32 + \frac{9}{5} \cdot 100 = 32 + 180 = 212°$ which is also correct. So you can use this formula to find Fahrenheit temperature corresponding to a Celsius temperature.

Both Fahrenheit and Celsius scales are quite convenient for everyday purposes but there is something that they lack related to the physical meaning of temperature. We learned that temperature corresponds to the internal kinetic energy of atoms and molecules. Kinetic energy could never be negative (recall $\frac{mv^2}{2}$ - mass m is always positive and square of any number is larger or equal to zero, so kinetic energy can not be negative). But Celsius and Fahrenheit temperatures could be negative. How is it possible? It is because choice of zero in both Celsius and Fahrenheit does not correspond to zero of kinetic energy. It makes sense to define a temperature scale with zero exactly corresponding to zero internal kinetic energy - the so-called **absolute zero of temperature**. Kelvin scale is defined this way: it begins from absolute zero and has the same increment as the Celsius scale. So Kelvin scale does not have negative temperatures: the absolute zero is at 0 Kelvins. In Celsius scale absolute zero is approximately at $-273°$ C. Conversely, the zero of Celsius scale (water freezing point) is 273 K (Kelvins are used without a ° sign). And 100° C (water boiling point) is 373 K. The general relation between Celsius and Kelvin scales is therefore

$$T = t_C + 273$$

where T is temperature in Kelvins and t_C is the corresponding temperature in Celsius.

Relating internal kinetic energy and temperature. Kelvin scale is especially convenient for relating temperature and average internal kinetic energy of atoms and molecules. This relation is as follows:

$$E_{kin} = \frac{3}{2} kT,$$

where T is temperature in Kelvins and $k = 1.38 \cdot 10^{-23}$ J/K is called the Boltzmann constant. Boltzmann constant is the coefficient between average internal kinetic energy of atoms and molecules and temperature. It is a very small number, so kinetic energies of atoms and molecules are very small. But this is what you should expect, because their mass is very tiny.

Homework

1. What is the temperature of a human body in the Kelvin scale?

2. Derive a general formula for calculating temperature in Kelvin from temperature in Fahrenheit. Use it to calculate 80° F in Kelvin.

3. Find the average speed of motion of oxygen molecules in air at temperature 300 K (this is often taken as the value of typical room temperature, because the number is nice). Mass of the oxygen molecule is $m_{O_2} = 2.7 \cdot 10^{-26}$ kg.

***4.** Assume there is exactly one oxygen molecule (from the last problem) in a room, flying back and forth between two walls. Estimate average force produced by this molecule on the walls. Take the distance between the walls to be 1 meter. How many molecules of the same kind are needed to produce force equal to 1 N? *Hint: You might want to refresh the relationships between momentum, impulse and force.*