GAS LAWS CONTINUED

APRIL 20, 2022

Theory recap

Gay-Lussac's law. Previously we discussed Boyle's law which deals with gas at a constant temperature. Today we will learn what happens if we change temperature. Let us begin with volume being fixed.

To imagine a fixed volume of gas, think of a sealed container (without a piston). Since we want to keep track of pressure, a pressure sensor can be attached to this container. In order to change the temperature of the gas we can put the whole container into hot water or some other environment of a given temperature. If we wait a bit, the container and the gas will reach the same temperature as the environment. Then by measuring temperature of the environment we know the temperature of the gas.

Assume that our gas in the closed container initially has pressure p_1 and temperature T_1 (in Kelvins). Then we increase its temperature to T_2 (also in Kelvins). French physicist Joseph Louis Gay-Lussac has discovered that pressure then also increases to p_2 according to the following relation:

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}.$$

An equivalent way of writing this relation is:

$$\frac{p}{T} = const \text{ for } V = const.$$

It is called Gay-Lussac's law.

For example, if at 300 K a gas has pressure 300 kPa then at 400 K and at the same volume it will have pressure 400 kPa, at 500 K pressure will become 500 kPa etc.

Why is it important that temperature in Gay-Lussac's law is measured in Kelvins? Imagine that instead of Kelvins we measured temperature in Celsius. As we know,

$$T = t + 273$$

where T is temperature in Kelvins and t is temperature in Celsius. So if we want to express temperature in Celsius in Gay-Lussac's law we would get

$$\frac{p_2}{p_1} = \frac{t_2 + 273}{t_1 + 273}$$

Does not look so nice, does it? We see that Kelvin scale allows us to write Gay-Lussac's law in a much simpler form.

Charle's law. The last of the three gas laws is the one for constant pressure. In practice one can easily keep the pressure constant by applying a constant force on the piston. We do not even have to apply force ourselves - atmospheric pressure is always acting on the piston from the outside. While keeping constant pressure we can heat the gas and measure how does it affect the gas volume. This way Charles's law was discovered (by French physicist and engineer Jacques Charles who was also the creator and first pilot of a manned hydrogen balloon). Charles's law says that volume changes proportionally to the temperature measured in Kelvins. If we compare gas at two states such that at one of them it has volume V_1 and temperature T_1 and in the other its' volume and temperature are V_2 and T_2, Charles's law reads:

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}.$$

An equivalent way of writing Charle's law is:

$$\frac{V}{T} = const \quad \text{for} \quad p = const.$$

For example, if at 300 K some gas had volume 300 cm^3 then at 400 K and at the same pressure it will have volume 400 cm^3, at 500 K volume will become 500 cm^3 etc. Let us try to visualize this law on a plot. The axes are temperature T versus volume V. T proportional to V corresponds to a straight line on this plot.

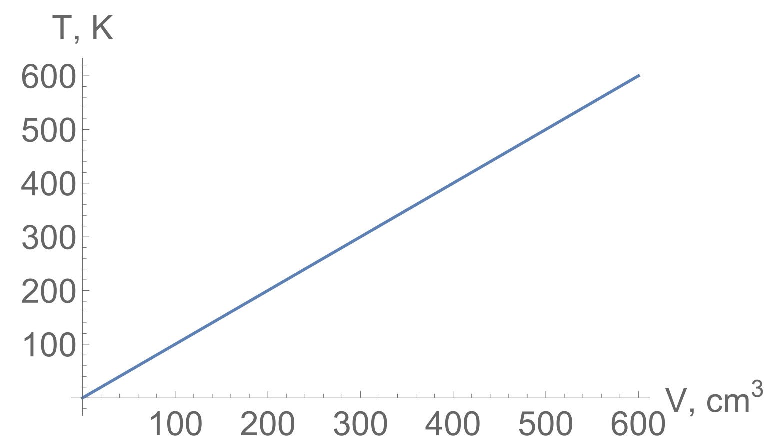

FIGURE 1. The line corresponds to constant pressure. Note that the temperature is in Kelvins

Note that this line goes through the point $(0, 0)$ so this plot predicts that at 0 K the gas will have zero volume. This is nonsense - no matter can be compressed to zero volume. The truth is, Charles's law and other gas laws are established for an **ideal gas** which is only an approximation to any real gas. In an ideal gas there is no interaction between its' molecules at all. In a real gas there is always some interaction but if the gas is rarefied enough it could be neglected. So the ideal gas is a good approximation for a gas of small density. However as gas becomes denser approximation by an ideal gas becomes worse. For a very low temperature and therefore small volume it is not a good approximation and Charles's law (and other laws) do not hold anymore.

Microscopic reasons behind gas laws. Gas laws have a clear interpretation in a microscopic description of a gas. Microscopically, a gas is a collection of molecules randomly flying around the container and occasionally colliding with each other and the container walls. Temperature tells us how fast the molecules are flying. Pressure tells us with what force do they act upon the walls of the container. And volume, for a fixed amount of gas, tells us how tightly (or, rather, loosely) the molecules are packed.

To understand Boyle's law in these terms, imagine we take a container of gas at some volume and pressure. If we decrease the volume of the gas while keeping temperature fixed, the gas becomes denser and collisions of gas particles with the walls happen more often. Each of these collisions still produces the same force on the wall as before (because this force depends on the speed of a gas molecule which has not changed since temperature is constant). And since there are more collisions, the total force exerted on the walls of the container is larger and therefore pressure is larger. This is what Boyle's law tells us: decreasing volume of a gas we get higher pressure.

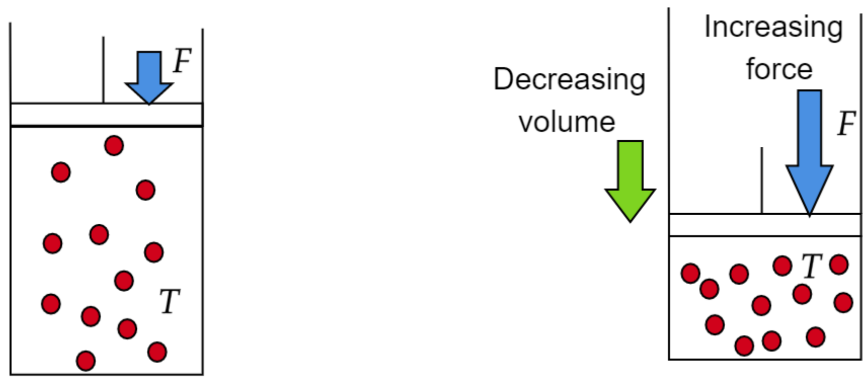

FIGURE 2. **Left figure:** Initially gas has some volume. Molecules collide with the piston and produce some force. An equal force is needed to balance it. **Right figure:** Volume of gas is decreased. Molecules are packed more densely and collide with the piston more often leading to the larger force. So, pressure of the gas increases.

Let us also understand Gay-Lussac's law. If we increase temperature, molecules have more kinetic energy. As a result, each individual collision of a molecule with the wall produces a larger force. If the volume stays the same the rate of collisions also grows because of increased speed of the molecules. Since there are more collisions per unit time and each collision is stronger, the total force increases. This means that pressure increases with growing temperature and constant volume. This is what Gay-Lussac's law tells us.

HOMEWORK

1. A cylinder is filled with gas. The pressure inside is 10000 Pa, the temperature is 20°C. We increase the temperature to 100°C. What happens to the pressure inside the cylinder? Calculate the new pressure.

2. Gas was heated from 27°C to 39°C but its pressure was maintained the same. By what percentage did the volume of the gas increase?

3. A gas has initial pressure 100 kPa, volume 100 cm^3 and temperature 27 °C. First, the gas is compressed at constant temperature so that its volume decreases two times. Then the volume of the container is fixed and the container is cooled down to -123 °C using liquid nitrogen. Find the final pressure of the gas.

***4.** Pressure of air in a bottle at 7°C is equal to the atmospheric pressure 100 kPa. How much does one need to heat the bottle so that a cork closing the bottle will be pushed out? Without heating the cork could be pulled out by force 10 N. Cross-section area of the cork is 2 cm^2.