IDEAL GAS PROCESSES. GRAPHICAL REPRESENTATION.

MAY 4, 2023

Theory recap

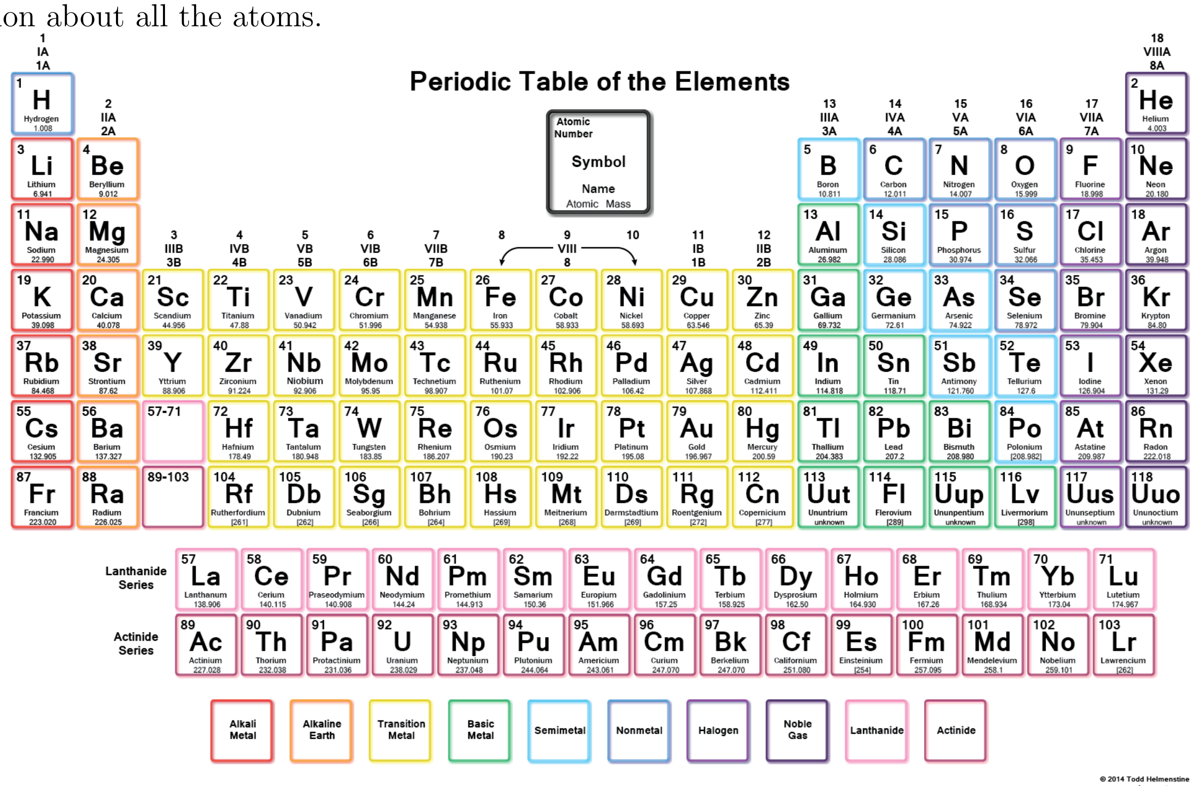
Molar mass. How do we know the amount of substance of some matter? It is really hard to resolve individual molecules, and there are too many of them (remember, the Avogadro number is $6 \cdot 10^{23}$) to count them one by one. But every molecule of a substance has the same mass, so $6.02 \cdot 10^{23}$ of these molecules have a particular mass which is characteristic of this substance. It is called molar mass and denoted by M. Mass is easy to measure and if we have m kilograms of substance with mass M, the amount of substance is

$$n = \frac{m}{M}.$$

Equation of state of ideal gas could be written in terms of mass and molar mass:

$$pV = \frac{m}{M}RT$$

Molar mass from periodic table. How do we know the molar mass of some substance? As we have seen, molar mass is related to mass of molecules of the substance. Mass of a molecule is equal to sum of masses of atoms comprising this molecule. And masses of atoms can be found in the periodic table of elements, which contains a lot of useful information about all the atoms.



There is a simple algorithm of finding molar mass from periodic table. First of all, we need to locate atomic mass in the periodic table: it is the lowest number in each cell. For example, for the first element - hydrogen (H) we can see the atomic mass is 1.008 which could be rounded to 1. This is exactly the molar mass of a hydrogen atom, measured in gram/mole. So, if we take 1 mole of hydrogen atoms, or $6 \cdot 10^{23}$ hydrogen atoms, their mass will be $M(\mathrm{H}) \cdot 1$ mole $= 1$ g. If we take 1 mole of carbon atoms, their mass will be $M(\mathrm{C}) \cdot 1$ mole $= 12$ g (find carbon C in the table above and verify that its' atomic mass is about 12).

Now, if we talk about molecules, molar mass is sum of molar masses of atoms building the molecules. For example, nitrogen molecule N_2 consists of two nitrogen atoms. Therefore, the molar mass of nitrogen molecules is twice the molar mass of nitrogen atoms:

$$M(\mathrm{N}_2) = 2 \cdot M(\mathrm{N}) = 2 \cdot 14 \text{ g/mole} = 28 \text{ g/mole}.$$

Let us do one more example. Consider a carbon dioxide molecule CO_2 which consists of a carbon atom and two oxygen atoms. From the periodic table we find that the molar mass of carbon atom C is 12 g/mole and that the molar mass of oxygen atom O is 16 g/mole. So we find molar mass of carbon dioxide:

$$M(\mathrm{CO}_2) = M(\mathrm{C}) + 2 \cdot M(\mathrm{O}) = 12 + 2 \cdot 16 \text{ g/mole} = 44 \text{ g/mole}.$$

Processes with ideal gas. Our ultimate goal is to understand how gases can be used in machines to extract work from heat. For that we need to learn a bit about processes that could happen to a gas and how to describe these processes conveniently. We already know that a gas in a given state is characterized by its pressure, volume and temperature and the amount of moles. Assuming that we fix amount of moles and don't change it, we only need to know two parameters, for example pressure and volume, to specify the state of the gas. Temperature then can be found using the ideal gas equation of state.

Graphically we can represent state of the gas as a point in $(p-V)$ coordinate plane: every point corresponds to some particular values of pressure and volume. For example, let us take one mole of some gas with pressure $p_0 = 101,339$ Pa and volume $V_0 = 0.0224$ m^3. This state is represented as a point on figure 1 (see below).

Knowing pressure and volume we could find temperature in this state:

$$p_0 V_0 = nRT_0 \implies T_0 = \frac{p_0 V_0}{nR} = 273.16\text{K}$$

Now let us decrease pressure of the gas while keeping the volume constant (processes at constant volume are called **isochoric**). In this process the gas will go through many intermediate states, all with the same volume. On our plot it will be represented by a continuous line with every point on it corresponding to some intermediate state. Constant volume means the volume coordinate is fixed, so this should be a vertical line (see figure 2 below). Its' endpoint will be at the final pressure which we will take to be $p_1 = 20,000$ Pa.

We could find the temperature T_1 in the final state with pressure p_1 by using either equation of state of ideal gas as above, or Gay-Lussac's law, which gives us

$$\frac{p_1}{T_1} = \frac{p_0}{T_0} \implies T_1 = T_0 \frac{p_1}{p_0} = 54 \text{ K}.$$

There is a possible caveat here, as 54 K is actually a very low temperature at which many gases, for example nitrogen or oxygen, become liquid and therefore can not be described by ideal gas equation of state. But there are gases which only condense at much lower

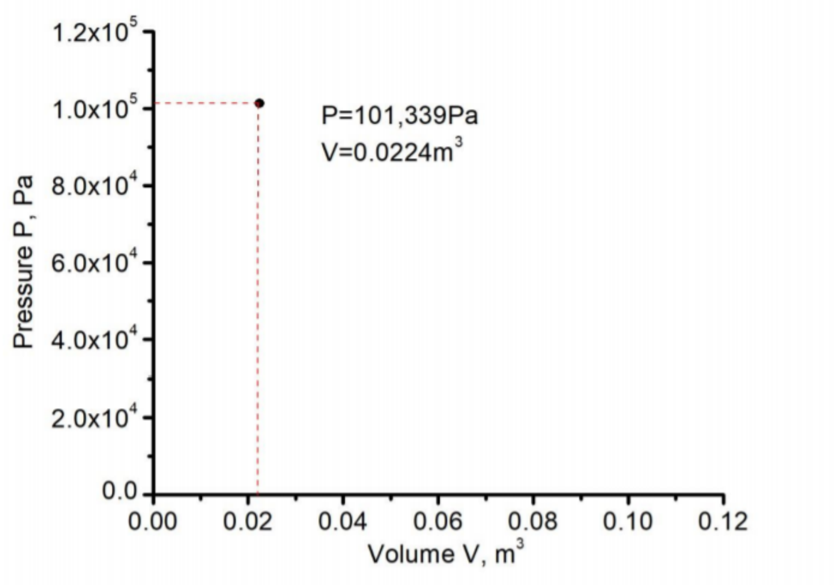


FIGURE 1. The point in $p - V$ coordinates represents the state in which the gas has pressure 101,339 Pa and volume 0.0224 m^3.

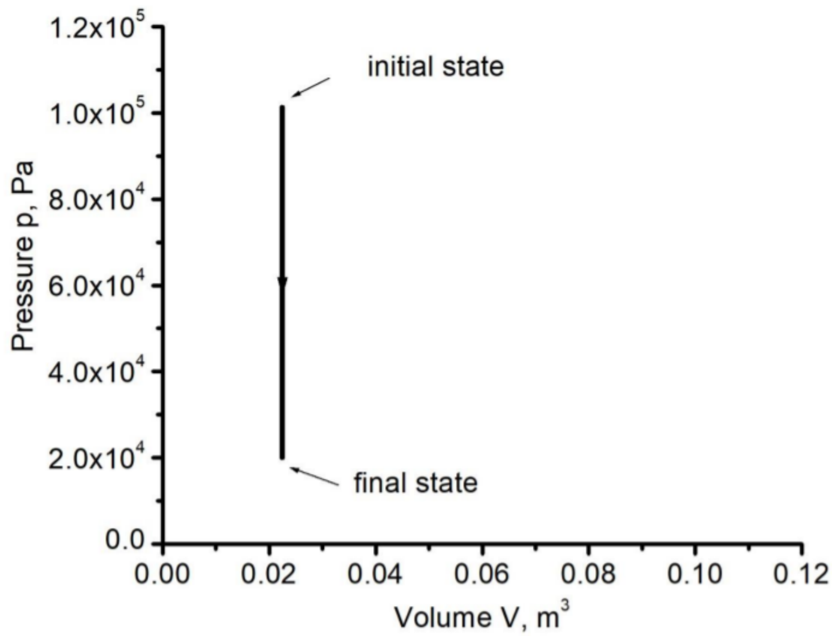


FIGURE 2. A vertical line in $p-V$ coordinates represents a process at constant volume, also called an isochoric process.

temperature, such as helium (at 4.2 K). So let us assume that here we work with helium and it behaves like an ideal gas at 54 K.

Now let us continue with our process. The next part will be done at constant pressure (processes at constant pressure are called **isobaric**) and increasing volume. Let us take the final volume to be $V_2 = 0.113$ m^3. Process at constant pressure is represented by a horizontal line on our plot (see figure 3).

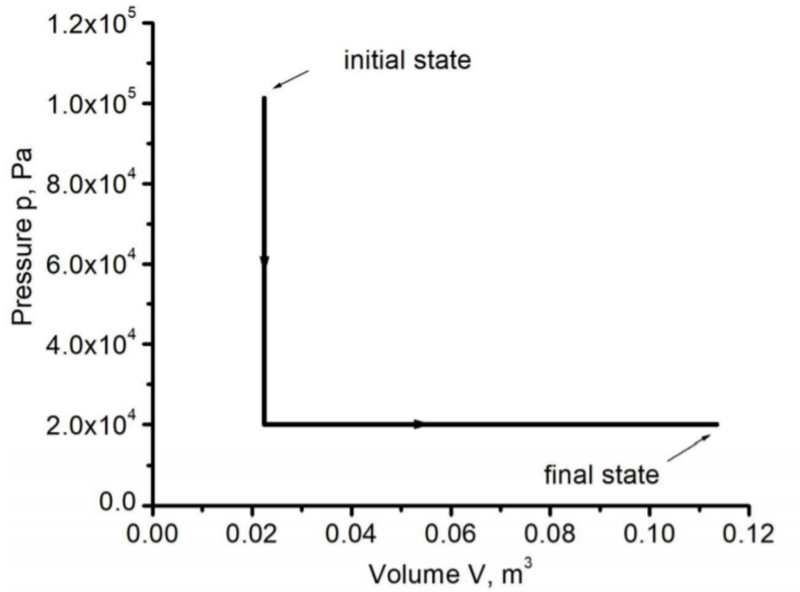


FIGURE 3. A horizontal line in $p - V$ coordinates represents a process at constant pressure, also called an isobaric process.

We can calculate temperature in the new final state this time using Charle's law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \implies T_2 = T_1 \frac{V_2}{V_1} = 273.16 \text{ K}.$$

So we have reached the same temperature as we had initially. We would like to return to the initial state because in order for us to be able to repeat the process again and again it should be cyclic. The simplest opportunity now is to compress the gas at constant temperature, or **isothermally**. As we discussed some time ago, on the $p - V$ plot the corresponding curve is hyperbola, as shown on figure 4. The equation of this curve could be found from equation of state of ideal gas:

$$pV = nRT \implies p = \frac{nRT}{V} = \frac{2270 \text{ J}}{V}.$$

HOMEWORK

1. Find molar mass of molecular oxygen O_2 using periodic table. Using it, find the mass of oxygen in a 10 liter cylinder if it has temperature T=13°C and pressure $P = 9 \cdot 10^6$ Pa (note that it is 90x the normal atmospheric pressure!). For how long can the oxygen in this cylinder sustain a scuba diver, if an average person needs to inhale about 2 grams of oxygen per minute?

2. There is a 1 liter bottle filled with water at 27°C. The water is liquid at this temperature because there is attracting force between the molecules. Imagine that we have

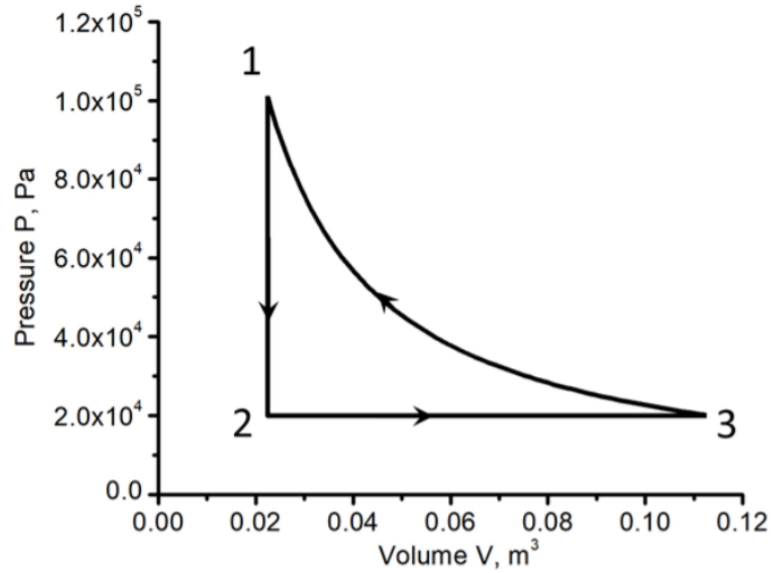


FIGURE 4. A hyperbola 3-1 in $p - V$ coordinates represents a process at constant temperature, also called an isothermal process.

suddenly "turned off" this attracting force. What is the pressure in the bottle now? *Hint: mass of 1 liter of water is 1 kg, molar mass of water is 18 grams/mole.*

3. Consider the following cyclic process performed with 3 moles of ideal gas. We start from pressure 10 kPa and volume 2 m^3 (point 1). Then we isobarically (which means keeping constant pressure) compress the gas until volume reaches 0.5 m^3 (point 2). Then at constant volume pressure is increased up to 40 kPa (point 3). After that keeping the pressure constant we bring the volume up to the initial value 2 m^3 (point 4). Finally pressure is isochorically (which means keeping constant volume) reduced and the gas comes back to point 1. Draw a diagram of this process in $p-V$ coordinates and find the temperature of the gas at points 1,2,3 and 4.

***4.** Any two of the three parameters of the gas (p, V, T) can be used as the coordinate axes, so the remaining one of the parameters will be found through the equation of state. It is useful to understand how to go between coordinates (p, V), (p, T) and (V, T). To get an idea about this, draw the cyclic process shown in Figure 4 in the coordinates (p, T) and (V, T).