Homework 5

Moment of inertia

Last class we discussed rotational inertia. We know that mass of an object determines its "regular" inertia. It means that the more massive the object is, the more force we need to accelerate it at a given rate. Is there a physical parameter which determines the rotational inertia? To answer this question let us try to calculate kinetic energy of a rotating object.

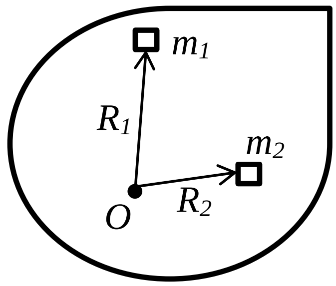


Figure 1.

Let us consider a rigid object of arbitrary shape which is rotating with an angular velocity $\boldsymbol{\omega}$ about an axis passing through point *O* (Figure 1) and perpendicular to the picture's plane. Let us "separate" the object to small pieces. The masses of the pieces ($\boldsymbol{m_1}$, $\boldsymbol{m_2}$,…$\boldsymbol{m_i}$) are , generally, not equal to each other. The distances between each piece and the rotation axis (point *O*) are $R_1, R_2 \ldots R_i$. To simplify Figure 1, I put only two of these pieces there. It is easy to understand that all the pieces will have same angular velocity, otherwise the object would be deforming during the rotation. Linear velocity V_1 of piece $\boldsymbol{m_1}$ is:

$$V_1 = R_1 \cdot \omega \qquad (1)$$

Kinetic energy E_{k1} of piece m_1 is:

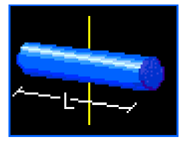
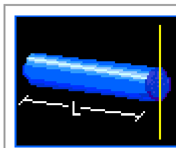
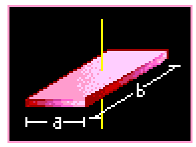
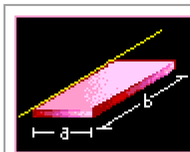
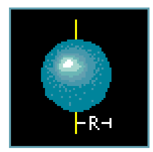
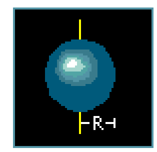
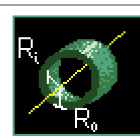
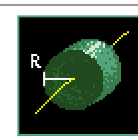
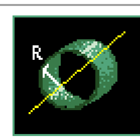
$$E_{k1} = \frac{m_1 \cdot V_1^2}{2} = \frac{m_1 \cdot R_1^2 \cdot \omega^2}{2} \qquad (2)$$

Total kinetic energy is the sum of kinetic energies of all the pieces:

$$E_{k\,total} = E_{k1} + E_{k2} + \cdots + E_{ki} = \frac{m_1 R_1^2 \omega^2}{2} + \frac{m_2 R_2^2 \omega^2}{2} + \cdots + \frac{m_i R_i^2 \omega^2}{2} \quad (3)$$

$$E_{k\,total} = (m_1 R_1^2 + m_2 R_2^2 + \cdots + m_i R_i^2)\frac{\omega^2}{2} = \frac{I \cdot \omega^2}{2} \quad (4)$$

In formula (4) we denoted the sum $m_1 R_1^2 + m_2 R_2^2 + \cdots + m_i R_i^2$ as *I*. This sum depends on the shape of the body, on the mass distribution in the body and the position of the rotation axis. Formula (4) is similar to very familiar expression for the kinetic energy for linear motion, where linear velocity is replaced by angular one and mass is replaced by parameter ***I*** which is called *moment of inertia*. It is the measure of the rotational inertia of the object. I have to stress that moment of inertia depends on the position of the rotation axis. Generally it is possible to calculate moment of inertia of any object, rotating about an arbitrary axis. Some of the moments of inertia are given in the table below:

slender rod:	axis through center	$I = \frac{1}{12} \cdot M \cdot L^2$		axis through end	$I = \frac{1}{3} \cdot M \cdot L^2$	
rectangular plane:	axis through center	$I = \frac{1}{12} \cdot M \cdot \left(a^2 + b^2\right)$		axis along edge	$I = \frac{1}{3} \cdot M \cdot a^2$	
sphere	thin-walled hollow	$I = \frac{2}{3} \cdot M \cdot R^2$		solid	$I = \frac{2}{5} \cdot M \cdot R^2$	
cylinder	hollow	$I = \frac{1}{2} \cdot M \cdot \left(R_i^2 + R_o^2\right)$		solid	$I = \frac{1}{2} \cdot M \cdot R^2$	
	thin-walled hollow	$I = M \cdot R^2$				

This table is taken from http://www.physics.uoguelph.ca/tutorials/

In the table above M is total mass of the object.

Problems:

1. Find kinetic energy of a car's wheel. The car is moving at a speed 100km/h. Assume that the wheel is a solid uniform cylinder with diameter of 50 cm, thickness 20cm and an average density of 1g/cm^3.

2. What are the similar features and differences between mass and moment of inertia?

3. A disk of mass *m* and radius *R* rolls downhill. The initial position of the disk center is at a height *h*. Find linear speed of the disk center as it reaches the ground.